Adaptive Model Following Systems for Flight Control and Simulation

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A method for the design of adaptive model following control systems has been developed using a hyperstability approach. The derivation of the adaptation algorithms is presented. The implementation of the algorithm is realized by a combination of linear filters with some positivity properties and of the multipliers which processes the model-plant error. The application of this method to the design of flight control and simulation systems is discussed. The feasibility and advantages of the procedure are illustrated by applying it to an aircraft longitudinal control problem.

Nomenclature

A_M = $n \times n$ matrix of model
A_P = $n \times n$ matrix of uncontrolled plant
B_M = $m \times n$ input matrix of model
B_P = $m_1 \times n$ input matrix of uncontrolled plant
$B_P^\dagger$ = $n \times m_1$ generalized inverse of $B_P(B_P^\dagger B_{P=I})$
$\beta(t)$ = $m_1 \times r$ linear matrix operator [defined in Eq. (39)]
C = $r \times n$ output matrix of uncontrolled plant
D = $m_1 \times n$ matrix [defined in Eq. (17)]
e = $n \times 1$ generalized error vector [defined in Eq. (14)]
$F_r(s)$ = transfer matrix of state variable filter [defined in Eq. (46)]
G = $n \times n$ matrix [defined in Eq. (47)]
$H_1, \tilde{H}$ = $n \times n$ positive definite (or semidefinite) matrices
K_M, K_P = $m_1 \times n$ control matrices
K_u = $m_1 \times m$ control matrix
$\Delta K_P(t,e)$ = $m_1 \times n$ adjustable part of matrix $K_P(t,e)$
$\Delta K_u(t,e)$ = $m_1 \times m$ adjustable part of matrix $K_u(t, e)$
$[\Delta K_P]_{ij}$ = i, j term of $\Delta K_P(t,e)$
$l(s)$ = polynomial in s
$L,\tilde{L},M,\tilde{M}$ = $m_1 \times m_1$ positive definite matrices
$M_q, M_{\dot{\alpha}}$ = longitudinal stability derivatives
$P,\tilde{P},Q$ = $n \times n$ positive definite matrices
q = pitch rate
R = $m \times m$ positive definite matrix
u_M = $m \times 1$ model input vector
u_P, u_{P1}, u_{P2} = plant input vectors [defined in Eq. (9)]
v = $n \times 1$ processed error vector [defined in Eq. (17)]
ν = air speed
x_M = $n \times 1$ state vector of model
x_P = $n \times 1$ state vector of plant
w, w_1 = $m_1 \times 1$ vectors [defined in Eq. (22)]
$Z(s)$ = $m_1 \times m_1$ transfer matrix [defined in Eq. (31)]
$Z_c(s)$ = Laplace transform of $\beta(t)$
α = angle of attack
μ_0, η = scalars
δe_M = elevator command input
δe_P = elevator deflection
δt_M = throttle command input
δt_P = throttle control
δ_P = flap deflection
ε = $r \times 1$ measurable part of vector e
$\Phi(v(\tau),t)$ = $m_1 \times n$ nonlinear time variable operator
$\Psi(v(\tau),t)$ = $m_1 \times m_1$ nonlinear time variable operator

Subscripts

M = model
P = plant
f = filtered values

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Introduction

FEEDBACK systems represent the conventional approach to implementation of the stability augmentation systems (S.A.S.) for aircraft. However, the development of new types of aircraft characterized by large variations in basic aircraft dynamics throughout the flight (e.g., V/STOL and variable geometry supersonic aircrafts) have indicated a need for a more versatile class of flight control systems. A first approach to the improvement of the basic feedback configuration of S.A.S. is the use of the linear model following control systems (L.M.F.C.).[1] Furthermore, the design of active simulation facilities can be formulated also as a model following control system problem.[2–4]

A model following control system has two given components: the reference model and the plant. In the case of a S.A.S., the reference model gives the desired response of the S.A.S. corresponding to certain handling qualities. In the case of an active simulation the reference model defines the input-output characteristics of the system to be simulated.

A L.M.F.C. system is usually represented as in Fig. 1a and described by the following equations

$$\dot{x}_M = A_M x_M + B_M u_M \tag{1}$$

$$\dot{x}_P = A_P x_P + B_P u_P \tag{2}$$

$$u_P = -K_P x_P + K_M x_M + K_u u_M \tag{3}$$

But a L.M.F.C. system also may be represented as in Fig. 1b where a certain similarity with a model reference adaptive system (M.R.A.S.) can be observed.[5]

A major problem for L.M.F.C. systems is to determine under what conditions "perfect model following" may be achieved with a bounded control function. Such conditions were established first by Erzberger[2] and they are expressed as

$$(I - B_P B_P^\dagger) B_M = 0 \tag{4}$$

$$(I - B_P B_P^\dagger)(A_M - A_P) = 0 \tag{5}$$

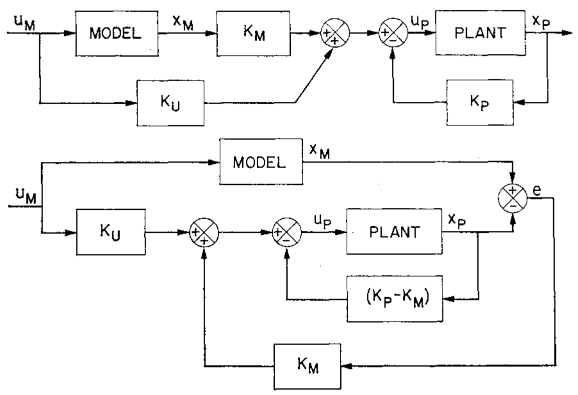


Fig. 1 Linear model following control system, (top) basic configuration, and (bottom) an equivalent representation.

New results for this problem were given recently by Curran[3] which introduces the concept of "equicontrolability" and gives a method for modifying slightly the model in order to satisfy the conditions for "perfect model following". As is shown in Refs. 2 and 3 the conditions for "perfect model following" are essentially related to the structure of A_M, B_M, A_P, B_P, and not to the values of the parameters.

Even when the conditions for "perfect model following" are satisfied, a major problem still remains; the performance of L.M.F.C. systems are high only if the parameters of the plant are known accurately and deviate only slightly from their nominal values, whereas in certain flight operations large variations of the plant parameters occur. An interesting approach in order to reduce the sensitivity of the L.M.F.C. systems to plant parameter variations using a trajectory sensitivity design is discussed by Winsor and Roy.[4] The authors conclude that: 1) the performance is high only if small variations of the parameters from their nominal values occur; 2) it is not possible to reduce the sensitivity of all the states to parameter variations; 3) increasing the sensitivity weighting factors in the performance index leads to degradation of the system response to state disturbances even for the nominal values of the parameters; 4) the dimension of the equivalent system which is optimized becomes considerably higher [$(2+p)n$ — where p is the number of variable parameters].

The analysis of the performance of various L.M.F.C. system designs leads to the conclusion that in order to assure high performance of a flight control system under all conditions, adaptive model following designs are generally required.

The technique of hyperstable M.R.A.S. as applied to improve the performance of the L.M.F.C. systems is discussed in the present paper. In developing the design of adaptive model following control systems (A.M.F.C.) four objectives were considered: 1) simple implementation; 2) strong stability characteristics; 3) high speed of adaptation; 4) systematic design method.

Two basic implementations of an A.M.F.C. system may be considered: a) parameter adaptation and b) signal synthesis adaptation (Fig. 2).

One can show that the two implementations are equivalent. For the A.M.F.C. system with parameter adaptation the plant input can be expressed by

$$u_P = -K_P(t,e)x_P + K_u(t,e)u_M + K_M x_M \tag{6}$$

where $K_P(t,e)$, $K_u(t,e)$ are time variable matrices depending on the generalized error e. But $K_P(t,e)$, $K_u(t,e)$ can be expressed as

$$K_P(t,e) = K_P - \Delta K_P(t,e) \tag{7}$$

$$K_u(t,e) = K_u + \Delta K_u(t,e) \tag{8}$$

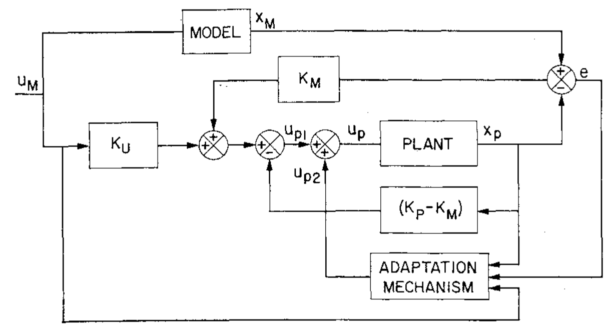


Fig. 2 Adaptive model following control system (signal synthesis adaptation).

where K_u, K_P are constant matrices designed for some specific plant parameter values. With this decomposition one can write that

$$u_P = u_{P1} + u_{P2} \tag{9}$$

where

$$u_{P1} = -K_P x_P + K_M x_M + K_u u_M \tag{10}$$

$$u_{P2} = \Delta K_P(t,e)x_P + \Delta K_u(t,e)u_M \tag{11}$$

The plant input u_{P2} is the contribution of the adaptive loop and the corresponding implementation as a signal synthesis adaptation is represented in Fig. 2. In this form the adaptation mechanism appears under the form of a supplementary feedback loop which improves the performance of the L.M.F.C. system.

In order to assure strong stability characteristics for the whole system, the hyperstability point of view was considered for developing the design method. The design method is based on theoretical and experimental results obtained in the synthesis of hyperstable M.R.A.S.[5–9] The resulting implementation is simple requiring only multipliers and linear filters. The practical difficulty which can appear in designing such a system when some of the states are not directly measurable is also discussed. Experimental results showing the improvement of performance which can be obtained with such a system in comparison with a L.M.F.C. system are presented.

The Adaptive Model Following Control System Equations

Consider an A.M.F.C. system with signal synthesis adaptation (Fig. 2) and described by the following equations

$$\dot{x}_M = A_M x_M + B_M u_M \tag{12}$$

$$\dot{x}_P = A_P x_P + B_P u_{P1} + B_P u_{P2} \tag{13}$$

$$e = x_M - x_P \tag{14}$$

$$u_{P1} = -K_P x_P + K_M x_M + K_u u_M \tag{15}$$

$$u_{P2} = \Delta K_P(t,e)x_P + \Delta K_u(t,e)u_M \tag{16}$$

The adaptation mechanism which generates the two matrices $\Delta K_u(t,e)\Delta K_P(t,e)$ must be designed in order to assure that

$$\lim_{t\to\infty} e(t) = 0$$

under certain conditions.

Based on previous results concerning this problem,[6,7] the adaptation mechanism is decomposed in two parts: a linear time invariant part which processes the generalized error e

$$v = De \tag{17}$$

and a second part which generates $\Delta K_P(t,e)$, $\Delta K_u(t,e)$ as a function of v

$$\Delta K_P(t,e) = \Delta K_P(t,v) = \Phi(v(\tau),t) \quad \tau \leq t \tag{18}$$

$$\Delta K_u(t,e) = \Delta K_u(t,v) = \Psi(v(\tau),t) \quad \tau \leq t \tag{19}$$

The hypotheses under which the design of the adaptive system is done are: 1) A_M,B_M,A_P,B_P belong to the class of matrices which verify the "perfect model following" conditions (4) and (5); 2) A_M,B_P are assumed to be time invariant during the adaptation process.

The method to be developed allows one to determine D, $\Phi(v(\tau),t)\Psi(v(\tau),t)$ in order that the whole system be asymptotically hyperstable—and therefore that

$$\lim_{t\to\infty} e(t) = 0$$

Combining the Eq. (12–16) and under the assumptions (4) and (5) one can write:

$$\dot{e} = (A_M - B_PK_M)e + B_P[B_P^{\dagger}(A_M - A_P) - K_M + K_P - \Delta K_P(t,e)]x_P + B_P[B_P^{\dagger}B_M - K_u - \Delta K_u(t,e)]u_M \tag{20}$$

Equation (20) together with Eqs. (17–19) define an equivalent feedback system described by the following equations

$$\dot{e} = (A_M - B_PK_M)e + B_Pw_1 \tag{21}$$

$$v = De \tag{17}$$

$$w = -w_1 = [\Phi(v(\tau),t) - B_P^{\dagger}(A_M - A_P) + K_M - K_P] \times x_P + [\Phi(v(\tau),t) - B_P^{\dagger}B_M + K_u]u_M \tag{22}$$

This equivalent feedback system can be partitioned into a linear time invariant part described by Eqs. (21) and (17) and a nonlinear time varying part described by Eq. (22). The design of an hyperstable A.M.F.C. system is transformed in this manner to the design of an asymptotically hyperstable nonlinear time varying feedback system. The resulting system will be hyperstable if it satisfies the conditions of Popov's hyperstability theorem.[6] Examining Popov's hyperstability theorem one finds that it is necessary first that the equivalent nonlinear part satisfies the inequality:

$$\eta(0,t_1) = \int_0^{t_1} v^T(t)w(t)dt \geq -\gamma_0^2 \tag{23}$$

On the basis of previous results in the synthesis of hyperstable M.R.A.S.[7] the following choices will lead to satisfaction of Eq. (23)

$$\Delta K_P(t,v) = \int_0^t \tilde{L}v(Qx_P)^T d\tau + Lv(Qx_P)^T + \Delta K_P(0) \tag{24}$$

$$\Delta K_u(t,v) = \int_0^t \tilde{M}v(Ru_M)^T d\tau + Mv(Ru_M)^T + \Delta K_u(0) \tag{25}$$

where $L,\tilde{L},Q,M,\tilde{M},R$, are positive definite matrices of appropriate dimension. The integral terms which contain v provide the memory of the adaptive mechanism (when $e\to 0$, $v\to 0$, respectively). The proportional terms on v are introduced in order to accelerate the reduction of the error e at the beginning of the adaptive process. They also are beneficial in the presence of parameters variations with zero steady state value and in the presence of state disturbances. This type of adaptation will be called "proportional + integral" (*PI*) adaptation.

Using the expressions (24) and (25) for $\Delta K_P(t,v)\Delta_u\Delta(t,v)$, the inequality (23) becomes

$$\begin{aligned}\eta(0,t_1) = &\int_0^{t_1} v^T\left[\int_0^t \tilde{L}v(Qx_P)^T d\tau - A_0\right]x_P dt \\ &+ \int_0^{t_1} v^T\left[\int_0^{t_1} \tilde{M}v(Ru_M)^T d\tau - B_0\right]u_M dt \\ &+ \int_0^{t_1}(v^TLv)(x_P^TQx_P)dt \\ &+ \int_0^{t_1}(v^TMv)(u_M^TRu_M)dt \geq -\gamma_0^2\end{aligned} \tag{26}$$

where

$$A_0 = B_P^{\dagger}(A_M - A_P) - K_M + K_P - \Delta K_P(0) \tag{27}$$

$$B_0 = B_P^{\dagger}B_M - K_u - \Delta K_u(0) \tag{28}$$

The last two integrals are greater or equal to zero because L,M,Q, and R, are positive definite matrices. For inequality (23) to hold, it is sufficient that each of the first two integrals in inequality (26) be greater than a negative finite constant. Using the properties of the positive definite matrices one can write the first integral as

$$I_1 = \int_0^{t_1} \tilde{v}_1^T\left[\int_0^t \tilde{v}\tilde{x}_P^T d\tau - \tilde{A}_0\right]\tilde{x}_P dt \tag{29}$$

where

$$\tilde{L} = L_1^TL_1;\ Q = Q_1^TQ_1;\ \tilde{v} = L_1v$$

$$\tilde{x}_P = Q_1x_P;\ A_0(Q_1)^{-1} = \tilde{A}_0$$

But the integral I_1 can be expressed also by

$$\begin{aligned}I_1 &= \sum_{i=1}^m \sum_{j=1}^m \int_0^{t_1} \tilde{v}_i\tilde{x}_{Pj}\left(\int_0^t \tilde{v}_i\tilde{x}_{Pj}d\tau - \tilde{a}_{ij}{}^0\right)dt \\ &= \frac{1}{2}\sum_{i=1}^m \sum_{j=1}^m\left[\left(\int_0^{t_1}\tilde{v}_i\tilde{x}_{Pj}d\tau - \tilde{a}_{ij}{}^0\right)^2 - (a_{ij}{}^0)^2\right] \geq -\frac{1}{2}\sum_{i=1}^m \sum_{j=1}^m (a_{ij}{}^0)^2\end{aligned}$$

The second integral in Eq. (26) verifies also such an inequality and therefore the inequality (26) is satisfied.

If the equivalent nonlinear part satisfies the condition (23) according to Popov's theorem[6] in order that the system be asymptotically hyperstable, the transfer matrix of the equivalent linear part [Eqs. (21) and (17)]

$$Z(s) = D(sI - A_M + B_PK_M)^{-1}B_P \tag{31}$$

must be strictly positive real. This implies first that $(A_M - B_PK_M)$ be a Hurwitz matrix and that D is given by

$$D = B_P^TP \tag{32}$$

where P is a positive definite matrix solution of the Lyapunov equation

$$(A_M - B_PK_M)^TP + P(A_M - B_PK_M) = -H \tag{33}$$

H being an arbitrary positive definite (or semidefinite) matrix. For the particular cases in which K_M is given by

$$K_M = B_P^T\tilde{P} \tag{34}$$

where $\tilde{P}$ is the solution of the algebraic Ricatti equation

$$\tilde{P}A_M + A_M^T\tilde{P} - \tilde{P}B_PB_P^T\tilde{P} + \tilde{H} = 0 \tag{35}$$

D can be chosen equal to K_M (the algebraic Ricatti equation is equivalent to a Lyapunov equation).

Equations (32, 24 and 25) give D, $\Phi[v(\tau),t]$, $\Psi[v(\tau),t]$ in order that the A.M.F.C. system described by Eq. (12–19) be asymptotically hyperstable. The implementation of the adaptation mechanism requires a gain matrix D which generates the vector v, followed by a set of modular structures of the type: multiplier—PI filter—multiplier; each of these structures corresponding to an adjustable term of K_P and K_u.

The Case of Partial States Measurement

When some of the states of a system are not directly measurable, the procedure used in the previous section to derive hyperstable adaptation algorithms can be also employed.

Consider a plant where only r states are available ($r < n$) and that the state equations are arranged in such a form that the available states correspond to the first r plant states. In this case the A.M.F.C. system is described by Eqs. (12–14) and the equations

$$\varepsilon = x_{M1} - x_{P1} = Ce \tag{36}$$

$$u_{P1} = -K_{P1}x_{P1} + K_M x_M + K_u u_M \tag{37}$$

$$u_{P2} = \Delta K_P(t,\varepsilon)x_{P1} + \Delta K_u(t,\varepsilon)u_M \tag{38}$$

$$v(t) = \int_0^t \beta(t-\tau)\varepsilon(\tau)d\tau;\ \mathscr{L}[\beta(t)] = Z_c(s) \tag{39}$$

For this case the vectors and the matrices are partitioned as shown below

$$x_P^T = |\underbrace{x_{P1}^T}_{r}, \underbrace{x_{P2}^T}_{n-r}| \qquad x_M^T = |\underbrace{x_{M1}^T}_{r}, \underbrace{x_{M2}^T}_{n-r}|$$

$$A_M = |\underbrace{A_{M1}}_{r}, \underbrace{A_{M2}}_{n-r}| \qquad A_P = |\underbrace{A_{P1}}_{r}, \underbrace{A_{P2}}_{n-r}|$$

$$K_M = |\underbrace{K_{M1}}_{r}, \underbrace{K_{M2}}_{n-r}| \qquad K_P = |\underbrace{K_{P1}}_{r}, \underbrace{0}_{n-r}|$$

From Eqs. (12–14, 37 and 38) one obtains

$$\begin{aligned}\dot{e} = (A_M - B_P K_M)e + [A_{M1} - A_{P1} - B_P K_{M1} + \\ B_P K_{P1}]x_{P1} + [A_{M2} - A_{P2} - B_P K_{M2}]x_{P2} + \\ [B_M - B_P K_u]u_M - B_P u_{P2}\end{aligned} \tag{40}$$

In order to apply the results from the previous section, it is sufficient that

$$A_{M2} - A_{P2} - B_P K_{M2} = 0 \tag{41}$$

which imply the satisfaction of the condition

$$(I - B_P B_P^{\dagger})(A_{M2} - A_{P2}) = 0 \tag{42}$$

In other words, A_{P1} must contain all the variable parameters while A_{P2} must have constant known parameters. Equation (41) always holds if the pairs (C,A_P) and (C,A_M) are completely observable ($x_{P1} = Cx_P$) and if A_M and A_P are expressed in a canonical form of the type given in Ref. 11.

Under the assumption (41), Eq. (40) becomes

$$\begin{aligned}\dot{e} = (A_M - B_P K_M)e + B_P \times \\ [B_P^{\dagger}(A_{M1} - A_{P1}) - K_{M1} + K_{P1} - \Delta K_{P1}(t,\varepsilon)] \times \\ x_{P1} + B_P[B_P^{\dagger}B_M - K_u - \Delta K_u(t,\varepsilon)]u_M\end{aligned} \tag{43}$$

Applying the same procedure as in the previous section, one finds that $\Delta K_u(t,\varepsilon)$ is given by Eq. (25) and $\Delta K_{P1}(t,\varepsilon)$ is given by

$$\Delta K_{P1}(t,\varepsilon) = \int_0^t Lv(Qx_{P1})^T d\tau + Lv(Qx_{P1})^T + \Delta K_{P1}(0) \tag{44}$$

and that the transfer matrix

$$Z(s) = Z_c(s)C(sI - A_M + B_P K_M)^{-1}B_P \tag{45}$$

must be strictly positive real. This condition allows one to determine $Z_c(s)$ under the assumption that the pairs (C,A_M) and (C,A_P) are c.o. and then to generate the vector v.

In order to generate v one can proceed in the following manner: a) $(n-r)$ integral transforms of ε are generated (assuming that C has r linear independent rows, if not $n - r_1$ integral transforms will be generated where r_1 is the number of linear independent rows in C) using a so called "state variable filter". A particular form of such a filter as applied to A.M.F.C. systems has the transfer matrix[9]

$$F_r(s) = l(s)I_r = (a_0/\sum_{i=0}^{n-r} a_i s^i)I_r \tag{46}$$

where the poles of $l(s)$ are different from the eigenvalues of A_M and A_P.

b) Because the pair (C,A_M) is c.o. for the $(n-r+1)r$ measurements available a set of n independent measurements are obtained.[11] These n measurements define a vector $\tilde{e}_f$ related to e_f (the equivalent filtered vector e) by

$$\tilde{e}_f = Ge_f \tag{47}$$

where G is constituted by the n linearly independent rows of the observability matrix.

c) A filtered vector v_f is given by

$$v_f = B_P^T PG^{-1}\tilde{e}_f \tag{48}$$

where P is the solution of Eq. (33) and $Z_c(s)$ is given by

$$Z_c(s) = B_P^T PG^{-1}\tilde{Z}_c(s) \tag{49}$$

where $\tilde{Z}_c(s)$ is defined as (ε_f denotes the filtered value of ε)

$$\tilde{e}_f(s) = \tilde{Z}_c(s)\varepsilon_f(s) \tag{50}$$

By using the filtered vector v_f the equations of the adaptive system are modified. The effects of such filters on the design of hyperstable M.R.A.S. in parameter tracking schemes is discussed in Ref. 9 but the results are also applicable to A.M.F.C. systems. Essentially the idea is that the generation of v_f is equivalent to constructing the vector $v = De$ for a system in which all the states are available and in which the input u_M is replaced by a filtered input u_{Mf} obtained through a filter with the transfer function $F_m(s) = l(s)I_m$. The system will be similar to the A.M.F.C. system described by Eqs. (12–19) if all the variables are replaced by their filtered values. Consequently $\Delta K_P(t,e_f)\Delta K_u(t,e_f)$ are given by Eqs. (24) and (25) in which the filtered values v_f,x_{Pf},u_{Mf} are used. The corresponding block diagram of such an A.M.F.C. system is represented in Fig. 3.

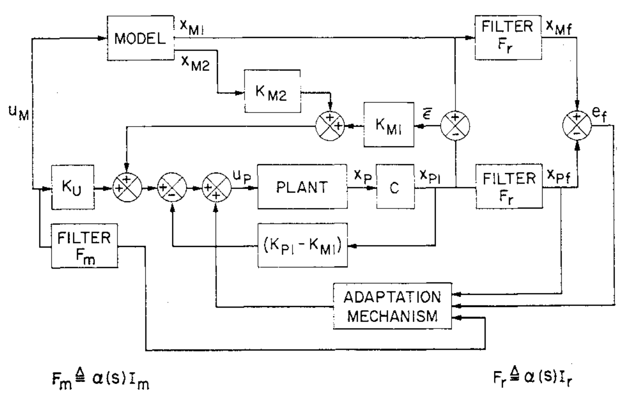


Fig. 3 Adaptive model following control system with some plant states available.

Design Example

The performances of A.M.F.C. systems designed by the method discussed in the previous sections can be illustrated by applying it to an aircraft control problem. The plant for this example represents the three degree of freedom linearized perturbation longitudinal state equations of a conventional subsonic aircraft (Convair C − 131 B) while the estimated dynamics of a large supersonic aircraft represents the model. The dynamics of the reference model is expressed by Eq. (1) where

$$x_M{}^T = [\theta_M, q_M, \alpha_M, v_M];\; u_M{}^T = [\delta e_M, \delta t_M]$$

$$A_M = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ 5.318 \times 10^{-7} & -0.4179 & -0.1202 & 2.319 \times 10^{-3} \\ -4.619 \times 10^{-9} & 1.0 & -0.7523 & -2.387 \times 10^{-2} \\ -0.5614 & 0.0 & 0.3002 & -1.743 \times 10^{-2} \end{bmatrix}$$

$$B_M = \begin{bmatrix} 0.0 & 0.0 \\ -0.1717 & 7.451 \times 10^{-6} \\ -0.0238 & -7.783 \times 10^{-5} \\ 0.0 & 3.685 \times 10^{-3} \end{bmatrix}$$

The plant is described by Eq. (2) where

$$x_P{}^T = [\theta_P, q_P, \alpha_P, v_P];\; U_P = [\delta e_P, \delta t_P, \delta z_P]$$

$$A_P = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ 1.4010 \times 10^{-4} & M_q + M_{\dot\alpha} & -1.9513 & 0.0133 \\ -2.5050 \times 10^{-4} & 1.0 & -1.3239 & -0.0238 \\ -0.5610 & 0.0 & 0.3580 & -0.0279 \end{bmatrix}$$

$$B_P = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ -5.3307 & 6.447 \times 10^{-3} & -0.2669 \\ -0.1600 & -1.155 \times 10^{-2} & -0.2511 \\ 0.0 & 0.1060 & 0.0862 \end{bmatrix}$$

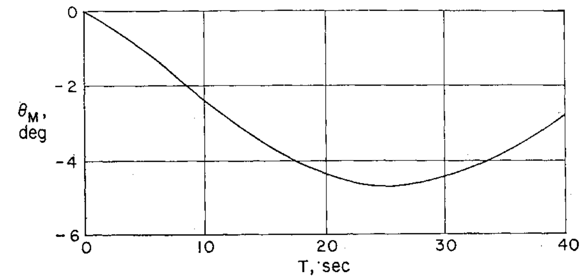
The nominal value for $M_q + M_{\dot\alpha}$ is −2.038 and it is assumed to vary from −0.558 to −3.558 (≈75% variations around the nominal value). The plant and the model satisfies the requirements for "perfect model following." The results obtained by simulation with the A.M.F.C. system are compared with the results obtained by Winsor and Roy for the same example using a L.M.F.C. system.[4]

The adaptive loop (Fig. 2) was superposed on the "perfect" L.M.F.C. designed for $M_q + M_{\dot\alpha} = -2.038$.[4] The adaptive mechanism was implemented using Eqs. (17, 24 and 25). The matrix D appearing in Eq. (17) was given by Eqs. (32) and (33) in which $H = 10^{-5}I$. The Lyapunov equation (33) was solved using the Jameson method[10] (one can observe that A_M is "ill conditioned").

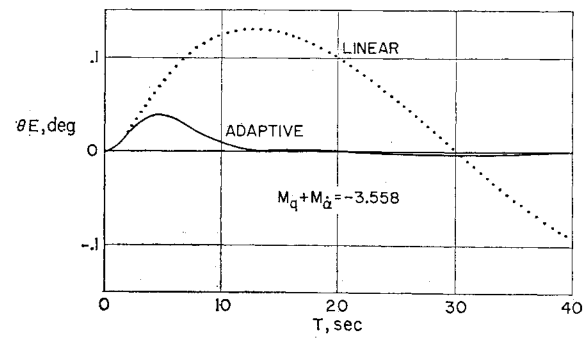
Three types of experiments were performed in order to evaluate the performance of the A.M.F.C. system by comparison with the L.M.F.C. system: 1) Comparison of the dynamic error response for various values of $M_q + M_{\dot\alpha}$ and *PI* adaptation (step input). 2) Same as at 1) but with proportional adaptation. 3) Comparison of the error response for an initial state error at the nominal value of $M_q + M_{\dot\alpha}$. The simulations were performed on a digital computer IBM 360/67 using the C.S.M.P. language.

Figures 4a, 5a, and 6a show the dynamic response of the model to unit step command inputs. For the nominal value of $M_q + M_{\dot\alpha}$ and no initial state error between plant and model states, the perfect model following prevails. The plant-model error ($-e$) for $M_q + M_{\dot\alpha} = -3.558$ is illustrated in Figs. 4b, 5b, 6b where the pitch angle error (θE), angle of attack error (αE) and air speed error (vE) are represented. The dotted curves represent the evolution of the error for the L.M.F.C. system mentioned above. The solid curves correspond to the A.M.F.C. system.

The results obtained for various values of $M_q + M_{\dot\alpha}$ for step input commands are summarized in Table 1 where the integral of the square error is tabulated. As one can observe

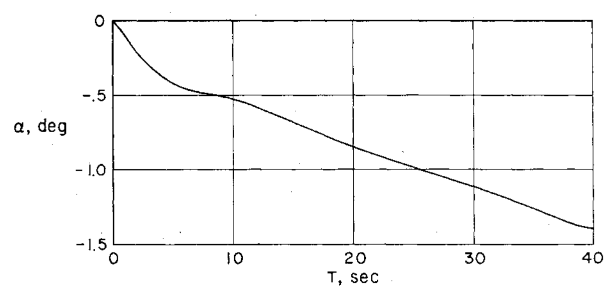


a) Pitch angle (model)

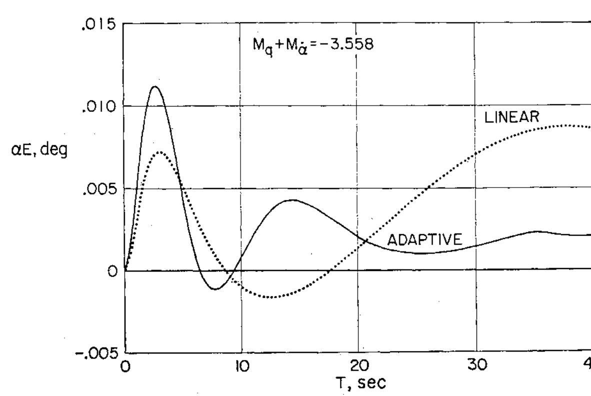


b) Plant-model pitch angle error
Fig. 4 Time response.

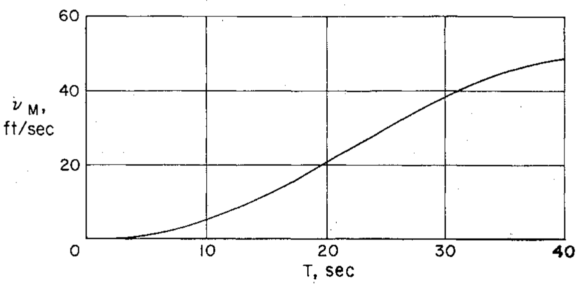
the variations of $M_q + M_{\dot\alpha}$ imply only the modification of the second column of K_P (realized by changing $\Delta K_P(t,e)$). In column B the initial value of $[\Delta K_P]_{12}$ $[\Delta K_P]_{22}$ was zero and $[\Delta K_P]_{32}$ was initialized at the correct value. The column C gives the performance of the A.M.F.C. system when the parameters $[\Delta K_P]_{12}$ $[\Delta K_P]_{22}$ $[\Delta K_P]_{32}$ were initialized at the



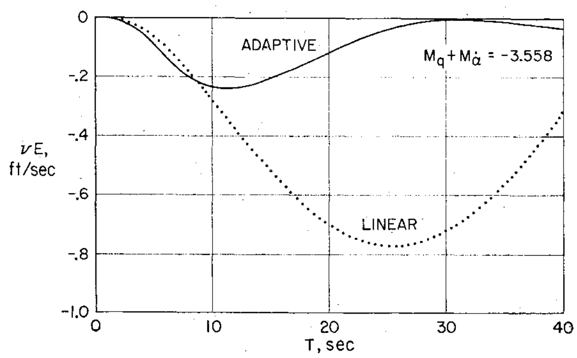
a) Angle of attack (model)



b) Plant-model angle of attack error
Fig. 5 Time response.



a) Air speed (model)



b) Plant-model air speed error
Fig. 6 Time response.

values obtained in the case B for $t = 40$ sec. Figure 7 illustrates the evolution of $[\Delta K_P]_{12}$, $[\Delta K_P]_{22}$ for the case $M_q + M_{\dot{\alpha}} = -3.558$ corresponding to column B in Table 1. One can observe that significant reduction of the plant-model error is obtained using an adaptive system even during the first step input.

The inability to reduce simultaneously all the plant-model states error during the adaptation process (compare Fig. 5b with Figs. 4b and 6b) arises from the fact that it is not possible to minimize independently each component of e. Using another choice for Φ, Ψ and D verifying the hyperstability

Table 1 Value of $\int_0^t e^T e dt$ for step inputs

	Linear model following control	Adaptive model following control (*PI* adaptation)	
	A	B	C
$M_q + M_{\dot{\alpha}}$	$t = 40$ sec	$t = 40$ sec	$t = 40$ sec
−3.558	12.047	0.621	0.20×10^{-2}
−0.558	11.843	0.750	0.23×10^{-2}

* Designed for $M_q + M_{\dot{\alpha}} = -2.038$.

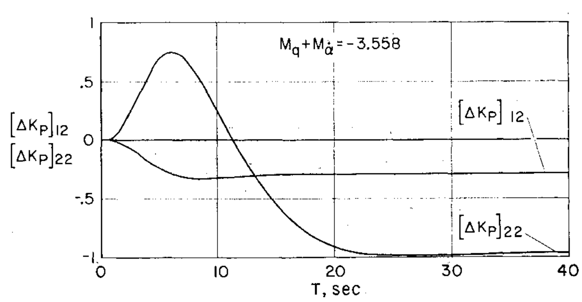
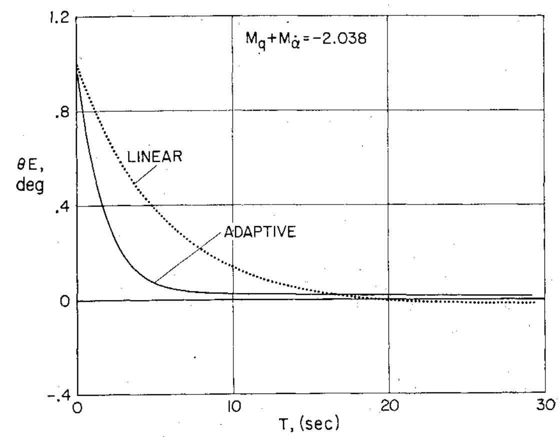


Fig. 7 Evolution of the parameters $[\Delta K_P]_{12}[\Delta K_P]_{22}$ during adaptation.

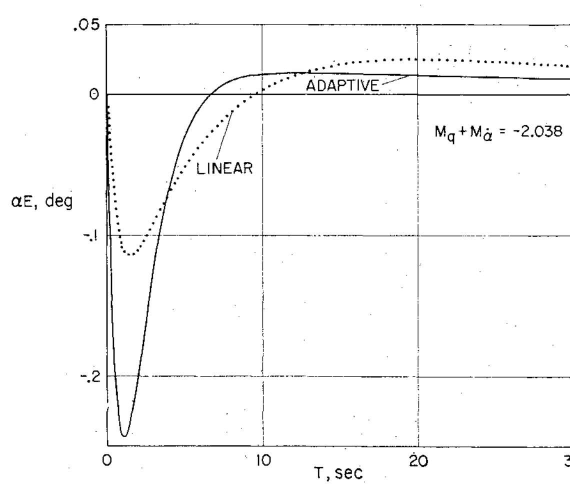
Table 2 Values of $\int_0^t e^T e dt$ for step inputs

	Linear model following control[a]	Adaptive model following control (*P* adaptation)	
$(M_q + M_{\dot{\alpha}})$	$t = 10$ sec	$t = 10$ sec	$t = 10$ sec[b]
−3.558	0.226	0.258×10^{-2}	0.086×10^{-2}
−0.558	0.278	0.243×10^{-2}	0.078×10^{-2}

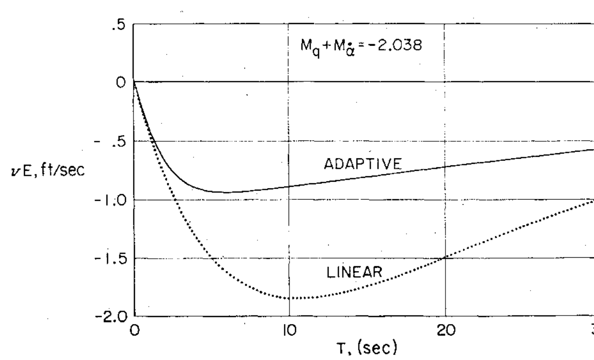
[a] Designed for $M_q + M_{\dot{\alpha}} = -2.038$.
[b] Corresponds to another choice of D.



a) Plant-model pitch angle error



b) Plant-model angle of attack error



c) Plant-model air speed error
Fig. 8 Elimination of a state disturbance.

Table 3 Values of $\int_0^t e^T e dt$ for an initial pitch attitude error at $(M_p + M_{\dot{\alpha}})_{nom} = -2.038$

t(sec)	6 sec	15 sec	21 sec	30 sec
Linear model following control	9.76	38.86	54.16	67.46
Adaptive model following control (*PI* adaptation)	4.70	11.61	14.98	18.44

conditions, an improvement of all plant-model states error can be obtained for about the same reduction of $\int_0^t {}^T e dt$.

The comparison between the L.M.F.C. system and the A.M.F.C. system having only proportional terms ($\tilde{L} \equiv \tilde{M} \equiv 0$) was also performed and the results are summarized in Table 2. The improvement of the performances is great but in contrast with the *PI* adaptation the error never becomes null.

The choice of the ratio between the proportional terms and the integral terms in the adaptation mechanism depends on the requirements. If a high reduction of the error in the initial moments are required the proportional term must be predominant, if a short time for reaching the steady-state values or the parameters is required the integral term must be predominant.

The last part of the experiments were concerned with the error response to an initial plant-model error of one degree in pitch angle for the nominal value of $M_q + M_{\dot{\alpha}}$. The results obtained with the A.M.F.C. (with *PI* adaptation) were compared with the best results obtained with a perfect L.M.F.C. (which corresponds to a system designed without sensitivity consideration[4]). The evolution of the components of the plant-model error are represented in Figs. 8a–c where the dotted curves correspond to the linear system and solid curves correspond to the adaptive system. These results are summarized in Table 3 where the values of $\int_0^t e^T e dt$ are tabulated. One can observe a reduction of $\int_0^t e^T e dt$ even at the "nominal values" when an adaptive system is used.

Conclusions

A general procedure has been developed for designing adaptive model following control systems using a hyperstability approach. The experimental results obtained by the simulation of an aircraft control problem have shown the feasibility and the performance of this type of A.M.F.C. system. Further studies will be concerned with the development of analytical methods for the optimization of the choice of the parameters of the adaptive mechanism related to some specific indexes of performance characterizing the adaptation process.

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